

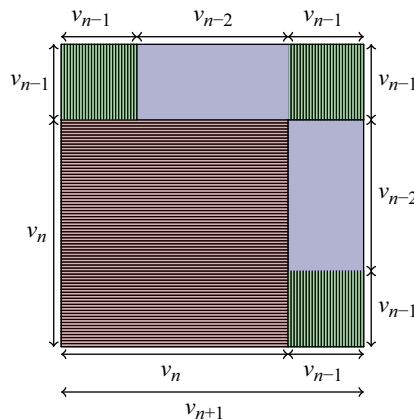
Proof Without Words: An Identity for a Recurrence Satisfied by the Fibonacci and Lucas Numbers

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Theorem. $v_{n+1} = v_n + v_{n-1} \implies v_{n+1}^2 = v_n^2 + 3v_{n-1}^2 + 2v_{n-1}v_{n-2}$.

Proof.



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Notice that the theorem applies to Fibonacci numbers ($F_{n+1} = F_n + F_{n-1}$, $F_0 = 0$, $F_1 = 1$) and Lucas numbers ($L_{n+1} = L_n + L_{n-1}$, $L_0 = 1$, $L_1 = 3$), since both follow the same recurrence relation.

Acknowledgment. The author wants to thank the referees and the editor for their comments to former versions of this proof without words which led to a completely new presentation.

Summary. We present a visual proof of an identity for three consecutive terms of a recurrence relation satisfied by the Fibonacci and Lucas numbers.

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